Kapitel V

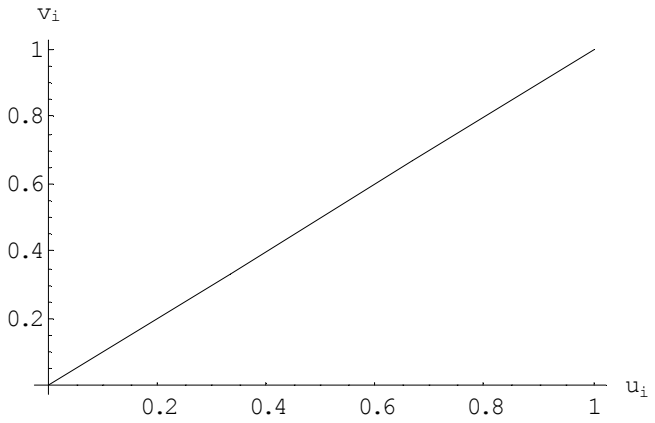
Konzentrationsanalyse
Lösungen

5. 1.

1. – 2.

M_1 :

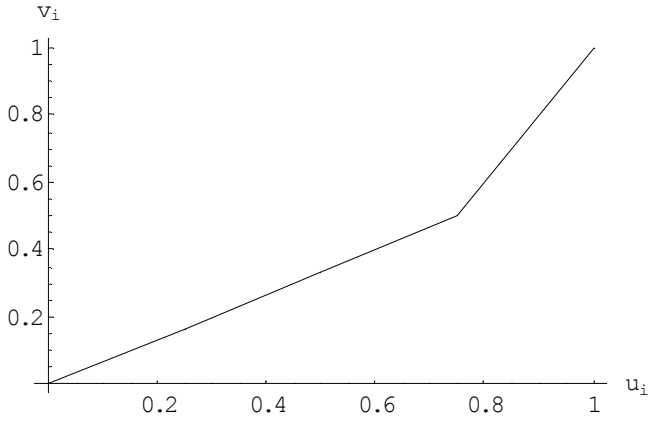
i	x_i	u_i	v_i
0	-	0	0
1	10	$\frac{1}{3}$	$\frac{1}{3}$
2	10	$\frac{2}{3}$	$\frac{2}{3}$
3	10	$\frac{3}{3}$	$\frac{3}{3}$



$$G = 1 - \left(\frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} \right) = 1 - \frac{1}{3} \left(\frac{1}{3} + \frac{1}{3} + \frac{1}{3} \right) = 0.$$

M_2 :

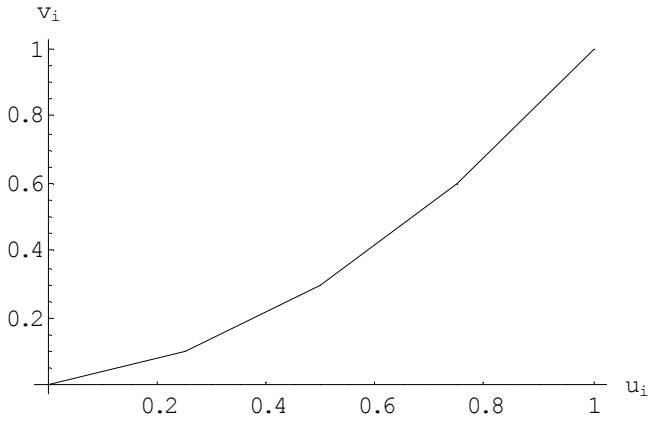
i	x_i	u_i	v_i
0	-	0	0
1	10	$\frac{1}{4}$	$\frac{1}{6}$
2	10	$\frac{2}{4}$	$\frac{2}{6}$
3	10	$\frac{3}{4}$	$\frac{3}{6}$
4	30	$\frac{4}{4}$	$\frac{6}{6}$



$$G = 1 - \frac{1}{3}\left(\frac{1}{6} + \frac{3}{6} + \frac{5}{6} + \frac{9}{6}\right) = 0.25.$$

M_3 :

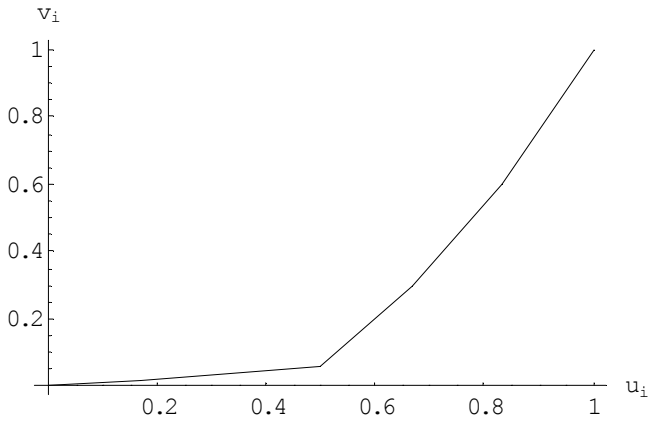
i	x_i	u_i	v_i
0	-	0	0
1	5	$\frac{1}{4}$	$\frac{1}{10}$
2	10	$\frac{2}{4}$	$\frac{3}{10}$
3	15	$\frac{3}{4}$	$\frac{6}{10}$
4	20	$\frac{4}{4}$	$\frac{10}{10}$



$$G = 1 - \frac{1}{4}\left(\frac{1}{10} + \frac{4}{10} + \frac{9}{10} + \frac{16}{10}\right) = 0.25.$$

M_4 :

i	x_i	u_i	v_i
0	-	0	0
1	1	$\frac{1}{6}$	$\frac{1}{50}$
2	1	$\frac{2}{6}$	$\frac{2}{50}$
3	1	$\frac{3}{6}$	$\frac{3}{50}$
4	12	$\frac{4}{6}$	$\frac{15}{50}$
5	15	$\frac{5}{6}$	$\frac{30}{50}$
6	20	$\frac{6}{6}$	$\frac{50}{50}$

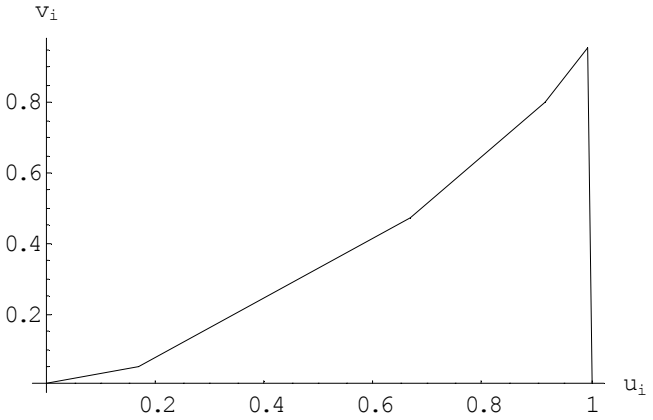


$$G = 1 - \frac{1}{6}\left(\frac{1}{50} + \frac{3}{50} + \frac{5}{50} + \frac{18}{50} + \frac{45}{50} + \frac{80}{50}\right) \approx 0.49.$$

5. 2.

Arbeitstabelle

i	g_i	G_i	H_i	h_i	$\sum_{j=1}^{i} h_j$	$\bar{x}_i$	$\bar{x}_i \cdot H_i$	$\sum_{j=1}^{i} \bar{x}_j \cdot H_j$	$\sum_{j=1}^{i} \bar{x}_j \cdot H_j \Big/ \sum_{i=1}^{p} \bar{x}_i \cdot H_i$
1	0	1000	2	0.167	0.167	600	1200	1200	0.05261
2	1000	2000	6	0.500	0.667	1600	9600	10800	0.47348
3	2000	3000	3	0.250	0.917	2500	7500	18300	0.80228
4	3000	5000	0.9	0.075	0.992	3900	3510	21810	0.95616
5	5000	-	0.1	0.008	1.000	10000	1000	22810	1.00000
			12.0	1.000			22810		



$$G = 1 - \frac{1}{n}\sum_{i=1}^{p} H_i \left(v_{i-1} + v_i\right)$$

$$= 1 - \sum_{i=1}^{p} h_i \left(v_{i-1} + v_i\right), \qquad 0 \le G \le 1 - \frac{H_p}{N}.$$

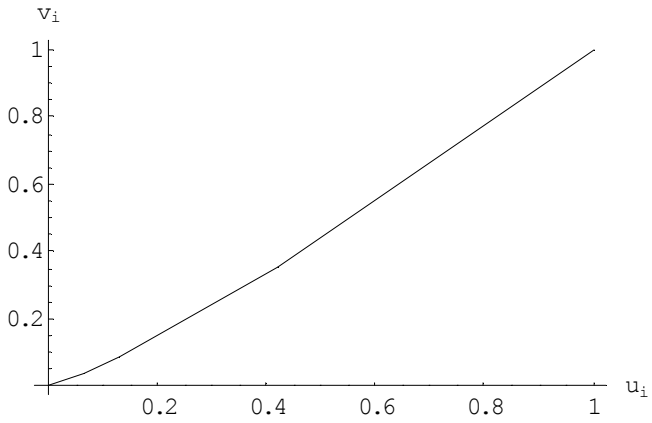
$$G = 1 - \frac{1}{12}\left(2 \cdot 0.05261 + 6 \cdot 0.52609 + 3 \cdot 1.27576 + 0.9 \cdot 1.75844 + 0.1 \cdot 1.95616\right) = 0.26106.$$

Die Konzentration der Einkommen ist somit insgesamt gesehen verhältnismäßig niedrig.

5. 3.

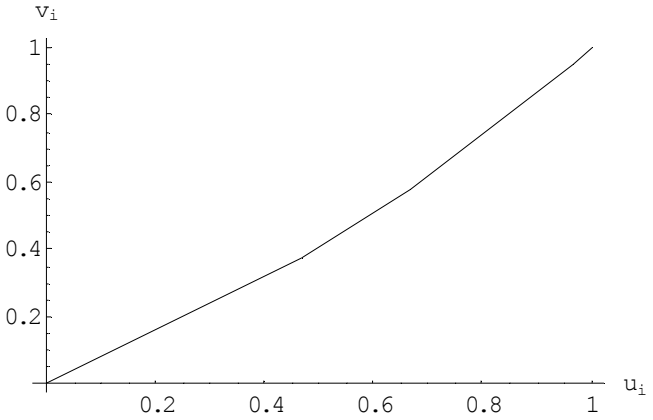
Arbeitstabelle
SC1

i	g_i	G_i	H_i	h_i	$\sum_{j=1}^{i} h_j$	m_i	$m_i \cdot H_i$	$\sum_{j=1}^{i} m_j \cdot H_j$	$\sum_{j=1}^{i} m_j \cdot H_j \Big/ \sum_{i=1}^{p} m_i \cdot H_i$
1	70	95	2	0.06452	0.06452	82.5	165.0	165.0	0.0382609869
2	95	115	2	0.06452	0.12904	105.0	210.0	375.0	0.0869565210
3	115	140	9	0.29032	0.41936	127.5	1147.5	1522.5	0.3530434780
4	140	170	18	0.58064	1.00000	155.0	2790.0	4312.5	1.0000000000
			31.0	1.00000			4312.5		



Arbeitstabelle
SC2

i	g_i	G_i	H_i	h_i	$\sum_{j=1}^{i} h_j$	m_i	$m_i \cdot H_i$	$\sum_{j=1}^{i} m_j \cdot H_j$	$\sum_{j=1}^{i} m_j \cdot H_j \Big/ \sum_{i=1}^{p} m_i \cdot H_i$
1	70	95	14	0.467	0.467	82.5	1155.0	1155.0	0.374089068
2	95	115	6	0.200	0.667	105.0	630.0	1785.0	0.578137651
3	115	140	9	0.300	0.967	127.5	1147.5	2932.5	0.949797570
4	140	170	1	0.033	1.000	155.0	155.0	3087.5	1.000000000
			30	1.000			3087.5		



2.

$$G = 1 - \frac{1}{n}\sum_{i=1}^{p} H_i \left(v_{i-1} + v_i \right)$$

SC1:

$$G = 1 - \frac{1}{31}\left(2 \cdot 0.0382609869 + 2 \cdot 0.125217507 + 9 \cdot 0.439999999 + 18 \cdot 1.439999999\right)$$

$$= 0.025582032 \approx 0.026$$

SC2:

$$G = 1 - \frac{1}{30}\left(14 \cdot 0.374089068 + 6 \cdot 0.952226719 + 9 \cdot 1.527935221 + 1 \cdot 1.949797570\right)$$

$$= 0.111605939 \approx 0.112..$$

3.

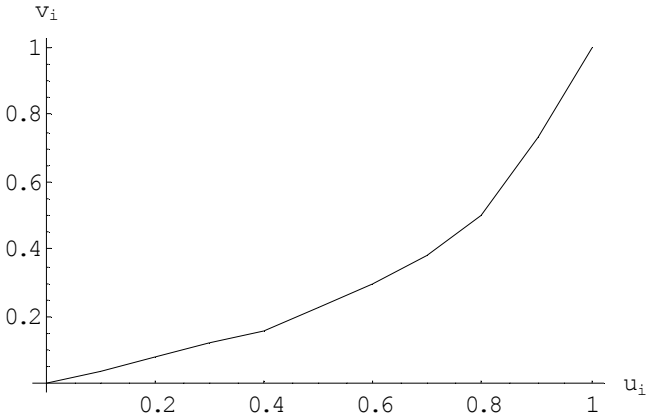
Es gibt höhere Konzentration der Sprungweiten im Verein SC2.

5. 4.

1.

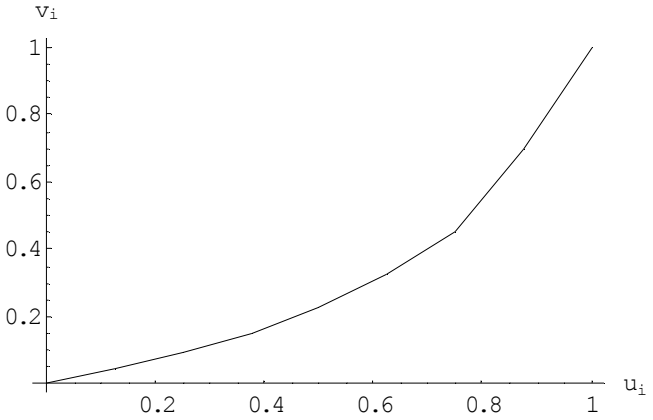
Arbeitstabelle (1996)

i	x_i	u_i	$\sum_{j=1}^{i} x_j$	v_i
0	-	0.0	-	0.00
1	40	0.1	40	0.04
2	40	0.2	80	0.08
3	40	0.3	120	0.12
4	40	0.4	160	0.16
5	70	0.5	230	0.23
6	70	0.6	300	0.30
7	80	0.7	380	0.38
8	120	0.8	500	0.50
9	230	0.9	730	0.73
10	270	1.0	1000	1.00
	1000			



Arbeitstabelle (2006)

i	x_i	u_i	$\sum_{j=1}^{i} x_j$	v_i
0	-	0.000	-	0.000
1	50	0.125	50	0.042
2	60	0.250	110	0.092
3	70	0.375	180	0.150
4	90	0.500	270	0.225
5	120	0.625	390	0.325
6	150	0.750	540	0.450
7	300	0.875	840	0.700
8	360	1.000	1200	1.000
	1200			



2.

$$G_{1996} = 1 - \frac{2}{10}\left(3.54 - 0.5\right) = 0.392,$$

$$G_{2006} = 1 - \frac{2}{8}\left(2.984 - 0.5\right) = 0.379.$$

3.
In der Branche war die Konzentration der Umsätze im Jahre 2006 etwas geringer als im Jahre 1996.

(Letzte Aktualisierung: 25.02.08)